



**Summer Holiday Home-work**  
**Class – XI, 2017-18, Mathematics**

- Let  $A = \{1, \{a, b\}\}$ . Is  $a \in A$ ? Justify.
- Are the following pair of sets equal? Give reasons.  
 $A =$  The set of letters in "ALLOY" and  $B =$  The set of letters in "LOYAL".
- If  $A = \{1, 2, \{3, 4\}, 5\}$  then which is incorrect and why. (i)  $\{3, 4\} \subset A$ , (ii)  $\{3, 4\} \in A$
- In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice And 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.
- For any two sets A and B prove by using properties of sets that:  
 $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$   
(It is denoted by  $A \Delta B$  and called Symmetric difference of two sets.)
- From 50 students taking examinations in Mathematics, Physics and Chemistry, each of the students has passed in at least one of the subjects, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 Mathematics and Chemistry and at most 20 Physics and Chemistry. What is the largest possible number that could have passed all three examinations.
- In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all three subjects. Find the no. of students that had taken (i) only chemistry (ii) only mathematics (iii) only physics (iv) physics and chemistry but not mathematics (v) mathematics and physics but not chemistry (vi) only one of the subjects (vii) at least one of three subject (viii) None of three subjects.
- Let  $f(x) = -|x|$ , then find Range of the function.
- Express  $\{(x, y) : y + 2x = 5 ; x, y \in W\}$  as the set of ordered pairs.
- Let  $R = \{(x, y) : x, y \in W, 2x + y = 8\}$  then  
(i) Write R as a set of ordered pairs. (ii) Find the domain and the range of R.
- Find the domain and the range of the following functions  $f(x) = \frac{1}{\sqrt{5-x}}$ .
- Define polynomial function. Draw the graph of  $f(x) = x^3$  find domain and range.
- Define least integer function. Draw its graph. Also find its domain and range.
- Find the principal solution of the equation  $\tan x = -\frac{1}{\sqrt{3}}$ .
- Show that  $\frac{\cos 5A + \cos 7A}{\sin 5A + \sin 7A} = \cot 6A$ .
- Solve:  $\sin \theta + \sin 3\theta + \sin 5\theta = 0$
- Prove that  $\cos \alpha + \cos \beta + \cos \gamma + \cos (\alpha + \beta + \gamma) = 4 \cos \frac{(\alpha+\beta)}{2} \cos \frac{(\beta+\gamma)}{2} \cos \frac{(\gamma+\alpha)}{2}$

18. Prove that  $\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$

19. Using mathematical induction, prove that the sum of the cubes of three consecutive natural no. is divisible by 9.

20. Using mathematical induction, prove that  $\frac{1}{3.6} + \frac{1}{6.9} + \frac{1}{9.12} + \dots + \frac{1}{3n(3n+3)} = \frac{n}{9(n+1)}$ .

21. Using mathematical induction, prove that  $n(n+1)(2n+1)$  is divisible by 6.

22. Using mathematical induction, show that  $2^{3n} - 1$  is divisible by 7

23. If  $\tan x = \frac{b}{a}$ , then find the value of  $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$ .

24. Find the modulus and argument of  $(-2 + 2\sqrt{3}i)$

25. Find the real values of  $x$  and  $y$  if  $(x + iy)(2 - 3i) = 4 + i$

26. Evaluate :  $\frac{i^{4n+1} - i^{4n-1}}{2}$ .

27. Solve the quadratic equation  $x^2 + (1 - 2i)x - 2i = 0$ .

28. If  $(\cos \theta - i \sin \theta)^2 = x - iy$ , prove that  $x^2 + y^2 = 1$

29. If  $z_1$  and  $z_2$  both satisfy  $z + \bar{z} = 2|z - 1|$  and  $\arg(z_1 - z_2) = \frac{\pi}{4}$ , then find  $\text{Im}(z_1 + z_2)$ .

30. If  $z_1$  and  $z_2$  are two non-zero complex numbers satisfying  $\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = 1$ , prove that  $\frac{z_1}{z_2}$  is purely imaginary.

31. What is polar form of the complex number  $(i^{25})^3$ ?

32. Solve:  $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

33. Solve graphically:  $x > 1 \cdot 5$ .

34. In drilling world's deepest hole it was found that the temperature  $T$  in degree Celsius,  $x$  km below the surface of earth was given by  $T = 30 + 25(x - 3)$ ,  $3 < x < 15$ . At what depth will the temperature be between  $155^\circ\text{C}$  and  $205^\circ\text{C}$ ?

35. Solve graphically  $2x + 3y \leq 150$ ,  $x + 4y \leq 80$ ,  $x \leq 15$ ,  $y \geq 0$  &  $x \geq 0$ .

36. Solve:  $\frac{x}{2x+1} \geq \frac{1}{4}$ ,  $\frac{6x}{4x-1} < \frac{1}{2}$ , and represent the solution on the number line.

37. Solve graphically the following system of inequalities

$$x + y \leq 4, 3x + y \geq 4, x + 5y \geq 4, x \leq 3, y \leq 3, x \geq 0, y \geq 0$$

40. If  $x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$ , then find the value of  $xy + yz + zx$ .

41. Show that  $2\sin^2 \beta + 4\cos(\alpha + \beta)\sin \alpha \sin \beta + \cos 2(\alpha + \beta) = \cos 2\alpha$ .

42. Solve the following system of inequalities:  $\frac{x+7}{x-8} > 2$ ,  $\frac{2x+1}{7x-1} > 5$ .

43. Prove that  $\cos \theta \cos 2\theta \cos 2^2 \theta \dots \cos 2^{n-1} \theta = \frac{\sin 2^n \theta}{2^n \sin \theta}$ , for all  $n \in \mathbb{N}$   $n \in \mathbb{N}$ .

44. Find the domain of each of the following functions

(i)  $f(x) = \frac{1}{\sqrt{1-\cos x}}$  (ii)  $f(x) = \frac{1}{\sqrt{x+|x|}}$  (iii)  $f(x) = x|x|$

45. Find the range of the following functions:

(i)  $f(x) = 1 + 3\cos 2x$  (ii)  $f(x) = 1 - |x - 2|$

46. Redefine the function  $f(x) = |x - 2| + |2 + x|$ ,  $-3 \leq x \leq 3$ .

47. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?

48. Show that  $\left| \frac{z-2}{z-3} \right| = 2$  represents a circle. Find its centre and radius.

49. Prove that  $\sin 4A = 4\sin A \cos^3 A - 4\cos A \sin^3 A$ .

50. Find the value of  $\left(1 + \cos \frac{\pi}{8}\right) \left(1 + \cos \frac{3\pi}{8}\right) \left(1 + \cos \frac{5\pi}{8}\right) \left(1 + \cos \frac{7\pi}{8}\right)$ .

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